Published by Institute of Physics Publishing for SISSA/ISAS

Received: *February 19, 2004*
Revised: *November 12, 2004*
Accepted: *December 21, 2004*

Gravitational wave spectrum in inflation with nonclassical states

Seoktae Koh and Sang Pyo Kim

Department of Physics, Kunsan National University
Kunsan 573-701, Korea
E-mail: kohst@ihanyang.ac.kr, sangkim@kunsan.ac.kr

Doo Jong Song

Korea Astronomy Observatory
Daejeon 305-348, Korea
E-mail: djsong@kao.re.kr

Abstract: The initial quantum state during inflation may evolve to a highly squeezed quantum state due to the interaction of the time-dependent parameter, $\omega_{\rm phys}(k/a)$, which may be the modified dispersion relation in trans-Planckian physics. This squeezed quantum state is a nonclassical state that has no counterpart in the classical theory. We have considered the nonclassical states such as squeezed, squeezed coherent, and squeezed thermal states, and calculated the power spectrum of the gravitational wave perturbation when the mode leaves the horizon.

Keywords: Cosmology of Theories beyond the SM, Large Scale Structure Formations, Physics of the Early Universe.

Contents

1. Introduction

Gravitational perturbations such as density perturbation and gravitational wave would have been created from quantum fluctuations during the inflation period when the wavelength of the perturbation was much smaller than the horizon size [1]. And if the wavelength stretches out to the Hubble radius during the inflation, the amplitude would be frozen out [2]. The power spectrum of the gravitational wave, if the initial state is an adiabatic vacuum state (Bunch-Davies vacuum) [3], gives a scale invariant one with $n_s = 1$ and $n_T = 0$ at the horizon crossing time, which is defined by [4]

$$P_s = A_s k^{n_s - 1}\,, \qquad P_T = A_T k^{n_T}\,, \tag{1.1}$$

where A_s and A_T are the amplitudes of scalar and tensor perturbations.

These gravitational wave and primordial density perturbations may have undergone a squeezing process due to a time-varying spacetime background [5]. If the hamiltonian is given by a free hamiltonian term $(\hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a})$ and an interacting term $(\hat{a}\hat{a} + \hat{a}^\dagger\hat{a}^\dagger)$ in the Heisenberg picture where $\hat{a}$ and $\hat{a}^\dagger$ are annihilation and creation operators, respectively, the squeezing occurs due to the interaction with a background. Although the squeezing can result from the interaction with the background spacetime, it is also possible to squeeze the vacuum state through a parametric interaction of the time-dependent dispersion relation at the subhorizon scale. This fact may be tested in the trans-Planckian physics.

The wavelength that corresponds to the present horizon size, when dated back to the early universe, might be smaller than the Planck scale [6, 7]. Though the Planck scale physics has not been completely understood, several ways were suggested to treat the trans-Planckian effects such as modifying dispersion relations, $\omega_{\rm phys}(k/a)$ [6, 7], considering non-vacuum initial state [8, 9], and modifying the space-time commutation relation motivated from the string theory [10]. In ref. [8], a squeezed vacuum state as an initial condition was used to calculate the consistency relations ($r \equiv A_T/A_s$) which is the ratio of the power spectra between scalar and tensor perturbations in inflation.

In this paper we shall consider in trans-Planckian physics whether the initial state may evolve to a squeezed quantum state due to the parametric interaction which comes from the modified dispersion relation. So the gravitational wave in the inflation period may exist, in general, in a squeezed quantum state, not the vacuum state. This squeezing process may slightly modify the power spectrum of the gravitational wave perturbations for superhorizon scales by the order of H^2/M^2 where M is the cutoff scale. We shall consider several types of nonclasscial states such as squeezed, squeezed coherent, and squeezed thermal states. The coherent states are in the borderline between quantum and classical states. The expectation value of the fields with respect to coherent states follow the classical equations of motion. So the coherent states may be a convenient tool to deal with the classicality of the quantum fluctuations. Further, it may be not excluded that the universe would have been very hot from the beginning. This means that thermal effects would play an important role during the history of the early universe. Thus, thermal states may have some relevance in dealing with the gravitational perturbations.

Nonclassical states that have no counterpart in classical theory are widely studied in quantum optics. Nonclassical states of light exhibit the features such as sub-poissonian photon statistics [11] and squeezing [12]. Quasiprobability distribution functions [13], which play a similar, but not exactly the same, role of a classical probability, are used to quantitatively measure the nonclassical behavior of states. A particularly useful distribution is the Glauber-Sudarshan P distribution that is defined in terms of the density operator [14]

$$\hat{\rho} = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha| \,, \tag{1.2}$$

where $|\alpha\rangle$ is a coherent state and α is a complex variable. If the P distribution of a state is more singular than a delta function and is not a positive definite, the state is classified as a nonclassical state [15]. The regularity condition for the quasiprobability distribution function is used to test whether states are either classical or nonclassical.

The organization of this paper is as follows. In section 2, we introduce the squeezed quantum states and quantize the gravitational wave perturbation in the inflation period. The squeeze parameter is found from the mode solution in the trans-Planckian theory. In section 3, we calculate the two-point correlation function in various nonclassical states and obtain the power spectrum of the gravitational wave perturbation for the superhorizon scale. We finally conclude and discuss in section 4.

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2. Quantization of scalar fields in de Sitter space with squeezed quantum state

The gravitational wave perturbation in the inflation period will be quantized using the two-mode squeezed state formalism. The metric of the spacetime in conformal time is given by

$$ds^2 = a^2(\eta)[-d\eta^2 + (\gamma_{ij} + h_{ij})dx^i dx^j], \tag{2.1}$$

where h_{ij} is a traceless, transverse perturbation and is assumed to satisfy $|h_{ij}| \ll \gamma_{ij}$. In the momentum space, h_{ij} is expanded as

$$h_{ij}(x) = \frac{1}{a}\int \frac{d^3k}{(2\pi)^{3/2}} \sum e_{ij}\mu_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{2.2}$$

where e_{ij} is a polarization tensor and μ_k is a complex function. Due to the reality condition of h_{ij}, one has $\mu_k^* = \mu_{-k}$.

The perturbed action for the gravitational wave is

$$\delta S = \int d^4x \frac{a^2}{2}[h'_{ij}h^{ij\prime} - F(\nabla, a)h_{ij}h^{ij}], \tag{2.3}$$

$$= \int d\eta d^3\mathbf{k}\left[\mu'_k\mu'_{-k} - \frac{a'}{a}(\mu'_k\mu_{-k} + \mu_k\mu'_{-k}) + \left(\left(\frac{a'}{a}\right)^2 - \omega^2(k)\right)\mu_k\mu_{-k}\right], \tag{2.4}$$

where a prime denotes the derivative with respect to the conformal time η, and $F(\nabla, a(\eta))$ is a function of spatial derivative, ∇, and the conformal time, and becomes $\omega(k)$ after Fourier transforming. The last term in the second line, a modified dispersion relation, takes into account the trans-Planckian effect. It represents a time-dependent non-linear property when the size of the perturbation mode is much smaller than the Planck scale,

$$\omega^2(k) = a^2\omega^2_{\text{phys}}\left(\frac{k}{a}\right), \tag{2.5}$$

but recovers the linear relation, $\omega \simeq k$, when the mode scale is greater than the Planck scale as in the standard inflationary model. From eq. (2.4), one obtains the conjugate momentum

$$\pi_k = \frac{\partial\mathcal{L}_k}{\partial\mu'_{-k}} = \mu'_k - \frac{a'}{a}\mu_k, \tag{2.6}$$

and the hamiltonian density

$$\mathcal{H}_k = \pi_k\mu'_{-k} + \pi_{-k}\mu'_k - \mathcal{L}_k,$$

$$= \pi_k\pi_{-k} + \omega^2(k)\mu_k\mu_{-k} + \frac{a'}{a}(\pi_k\mu_{-k} + \pi_{-k}\mu_k). \tag{2.7}$$

The hamiltonian density leads to the equation of motion

$$\mu''_k + \left(\omega^2(k) - \frac{a''}{a}\right)\mu_k = 0. \tag{2.8}$$

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To quantize the fields we introduce two linear time-dependent operators [16, 17]

$$\hat{a}_k(\eta) = -i\left(\varphi_k^{*\prime}(\eta) - \frac{a'}{a}\varphi_k^*(\eta)\right)\hat{\mu}_k + i\varphi_k^*(\eta)\hat{\pi}_k\,,$$
$$\hat{a}^\dagger_{-k}(\eta) = i\left(\varphi_k'(\eta) - \frac{a'}{a}\varphi_k(\eta)\right)\hat{\mu}_k - i\varphi_k(\eta)\hat{\pi}_k\,. \tag{2.9}$$

Here we treated $\hat{\mu}_k$ and $\hat{\pi}_k$ as Schrödinger operators. These operators in eq. (2.9) satisfy the quantum Liouville-von Neumann equation [16]–[19]

$$i\frac{\partial}{\partial\eta}\hat{a}_k(\eta) + [\hat{a}_k(\eta), \hat{\mathcal{H}}_k(\eta)] = 0\,, \tag{2.10}$$

and the same equation holds for $\hat{a}_k^\dagger$. It should be remarked that the eigenstates of $\hat{a}_k$ and $\hat{a}_k^\dagger$ are exact solutions of the time-dependent Schrödinger equation, when φ_k is a complex solution to the equation of motion

$$\varphi_k'' + \left(\omega^2(k,\eta) - \frac{a''}{a}\right)\varphi_k = 0\,. \tag{2.11}$$

The commutation relation of the fields in the momentum space at equal times

$$[\hat{\mu}_k, \hat{\pi}_{k'}] = i\delta(k+k')\,, \tag{2.12}$$

leads to the usual commutation relation

$$[\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta(k-k')\,, \tag{2.13}$$

when the Wronskian condition

$$\varphi_k\varphi_k^{*\prime} - \varphi_k^*\varphi_k' = i \tag{2.14}$$

is imposed. We may thus interpret the $\hat{a}_k$ and $\hat{a}^\dagger_{-k}$ as a time-dependent annihilation and creation operator. These operators do not, however, diagonalize the hamiltonian (2.7), but provide the exact quantum states of the time-dependent Schrödinger equation [18].

There are two ways of squeezing quantum states. First, we choose some preferred complex solution to eqs. (2.11) and (2.14), which may be chosen based on, for instance, the minimum particle creation postulate [20] or the minimum uncertainty [17]. Then a general solution to eqs. (2.11) and (2.14) is the superposition

$$\varphi_{ks}(\eta) = u_0\varphi_k(\eta) + v_0\varphi_k^*(\eta)\,, \tag{2.15}$$

with

$$|u_0|^2 - |v_0|^2 = 1\,. \tag{2.16}$$

Here the subscript s denotes a squeezed state. We may write u_0 and v_0 as squeeze parameters

$$u_0 = \cosh r\,, \qquad v_0 = e^{-i\phi}\sinh r\,. \tag{2.17}$$

Second, a given state may be squeezed due to a parametric interaction, here, time-dependent spacetime background. The preferred solution φ_k at η can be written as the superposition of φ_k and φ_k^* at a different conformal time η_0. Note that $\varphi_k(\eta)$ and $\varpi_k(\eta)(\equiv \varphi'(\eta) - \frac{a'}{a}\varphi(\eta))$ satisfy the Hamilton equations. Then the two functions

$$\begin{aligned} u_k(\eta,\eta_0) &= i[\varpi_k(\eta)\varphi_k^*(\eta_0) - \varphi_k(\eta)\varpi_k^*(\eta_0)]\,, \\ v_k(\eta,\eta_0) &= -i[\varpi_k(\eta)\varphi_k(\eta_0) - \varphi_k(\eta)\varpi_k(\eta_0)]\,, \end{aligned} \tag{2.18}$$

satisfy eq. (2.11) at η and η_0, and have the initial value $u_k(\eta_0,\eta_0) = 1$ and $v_k(\eta_0,\eta_0) = 0$. The inverse transformation of eq. (2.18) is found to be

$$\begin{aligned} \varphi_{ks}(\eta) &= u_k(\eta,\eta_0)\varphi_k(\eta_0) + v_k(\eta,\eta_0)\varphi_k^*(\eta_0)\,, \\ \varpi_{ks}(\eta) &= u_k(\eta,\eta_0)\varpi_k(\eta_0) + v_k(\eta,\eta_0)\varpi_k^*(\eta_0)\,. \end{aligned} \tag{2.19}$$

In fact, $\varphi_{ks}(\eta)$ and $\varpi_{ks}(\eta)$ are solutions to Hamilton equations with the initial data $\varphi_k(\eta_0)$ and $\varpi_k(\eta_0)$.

In the both cases, substituting eqs. (2.15) and (2.19) into eq. (2.9), we obtain the Bogoliubov transformation

$$\hat{a}_{ks}(\eta) = u^*\hat{a}_k(\eta) - v^*\hat{a}^\dagger_{-k}(\eta)\,, \tag{2.20}$$

$$\hat{a}^\dagger_{-ks}(\eta) = u\hat{a}^\dagger_{-k}(\eta) - v\hat{a}_k(\eta)\,. \tag{2.21}$$

In terms of the squeeze operator [21]

$$\hat{S}_k(z,\eta) = \exp\left[z_k\hat{a}_k^\dagger(\eta)\hat{a}^\dagger_{-k}(\eta) - z_k^*\hat{a}_k(\eta)\hat{a}_{-k}(\eta)\right]\,, \tag{2.22}$$

where $z_k = r_k e^{i\phi_k}$, the Bogoliubov transformation can also be written as

$$\hat{a}_{ks}(\eta) = \hat{S}_k(z,\eta)\hat{a}_k(\eta)\hat{S}_k^\dagger(z,\eta)\,, \tag{2.23}$$

$$\hat{a}^\dagger_{-ks}(\eta) = \hat{S}_k(z,\eta)\hat{a}^\dagger_{-k}(\eta)\hat{S}_k^\dagger(z,\eta)\,. \tag{2.24}$$

The squeeze operator $\hat{S}(z,\eta)$ maps a number state into a squeezed number state which is an eigenstate of the squeezed number operator, $\hat{N}_{ks}(\eta) \equiv \hat{a}^\dagger_{ks}(\eta)\hat{a}_{ks}(\eta)$,

$$|n,\eta\rangle_s = \hat{S}(z,\eta)|n,\eta\rangle\,, \tag{2.25}$$

where

$$|n,\eta\rangle_s = |n,z;\mathbf{k},-\mathbf{k},\eta\rangle\,. \tag{2.26}$$

We now solve the mode equation (2.11) in each region during the inflation period. Consider an exponential inflation with the scale factor given by

$$a(\eta) = -\frac{1}{H\eta}\,. \tag{2.27}$$

When momenta k are greater than the comoving cutoff scale, $k_c \equiv aM$, where M is the proper cutoff mass scale, the dispersion relation $\omega(k)$ shows the non-linear property. Several forms of $\omega_{\rm phys}(k/a)$ in this region are suggested in literature [6]–[10]. To solve the equation in this region, we assume that the adiabatic approximation holds, ($|\omega'| \ll \omega^2$), and get the WKB-type solution

$$\varphi_k(\eta) = \frac{1}{\sqrt{2\omega}} e^{-i\int \omega(\eta')d\eta'} \,. \tag{2.28}$$

If $\omega_{\rm phys}(k/a)$ is given by [6]

$$\omega_{\rm phys}\left(\frac{k}{a}\right) = M \tanh^{1/p}\left[\left(\frac{k}{aM}\right)^p\right], \tag{2.29}$$

where p is an arbitrary coefficient, the condition of the adiabatic approximation requires that $H/M \ll 1$. In fact, the small amplitude of cosmic microwave background fluctuations constrains $H/M \leq 10^{-5}$ while our present Hubble scale crossed the horizon during inflation [22]. The mode crosses the cutoff scale when $k = k_c$. If the wavelength of the mode is larger than the cutoff length scale, then the dispersion relation becomes linear in the physical wavenumber, $\omega(k) \simeq k$, and leads to the exact solution

$$\varphi_k(\eta) = \left(1 - \frac{i}{k\eta}\right)\frac{e^{-ik\eta}}{\sqrt{2k}} \,. \tag{2.30}$$

The variances of the field and its conjugate momentum are calculated with the solutions (2.28) and (2.30) to see whether the squeezing occurs during the expansion of the universe. In the trans-Planckian era, the variances with respect to the vacuum state are

$$\Delta\mu_k^2 \equiv \langle \hat{\mu}_k \hat{\mu}_{-k}\rangle = \frac{1}{2\omega}\,, \tag{2.31}$$

$$\Delta\pi_k^2 \equiv \langle \hat{\pi}_k \hat{\pi}_{-k}\rangle = \frac{\omega}{2}\left(1 + \frac{1}{\omega^2\eta^2}\right), \tag{2.32}$$

and

$$\Delta\mu_k^2 \Delta\pi_k^2 = \frac{1}{4}\left(1 + \frac{1}{\omega^2\eta^2}\right). \tag{2.33}$$

Note that the uncertainty of the gravitational wave field h_{ij} and its conjugate momentum are related as

$$\Delta h_k^2 \equiv \frac{1}{a^2}\Delta\mu_k^2\,, \qquad \Delta\Pi_k^2 \equiv a^2 \Delta\pi_k^2\,. \tag{2.34}$$

If ω is given in the form of eq. (2.29), we can approximate $\omega \simeq k_c = aM$ for $k \gg k_c$. Then the uncertainty in the field, $\Delta\mu_k$, decreases as the universe expands, whereas $\Delta\pi_k$ increases. However, these have the minimum uncertainty as long as $H/M \ll 1$. So we can see that the squeezing occurs due to not only the expansion of the universe but the time-dependent parameter ω in the trans-Planckian era. On the other hand, for $k < k_c$, the variances of the field and momentum with respect to the squeezed vacuum state are

$$\Delta\mu_k^2 = \frac{1}{2k}\left(1 + \frac{1}{k^2\eta^2}\right), \tag{2.35}$$

$$\Delta\pi_k^2 = \frac{k}{2}\,, \tag{2.36}$$

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and

$$\Delta\mu_k^2 \Delta\pi_k^2 = \frac{1}{4}\left(1 + \frac{1}{k^2\eta^2}\right). \tag{2.37}$$

The modes whose wavelengths stay inside the horizon ($k\eta \gg 1$) have the minimum uncertainty. For the superhorizon scales ($k\eta \ll 1$), however, both the variance of the field, $\Delta\mu_k$, and the uncertainty $\Delta\mu_k\Delta\pi_k$ increase with the conformal time. The scalar field in inflation can be described classically if the uncertainty $\Delta\mu_k\Delta\pi_k$ is much larger than the minimum uncertainty [23]. So in this sense the gravitational wave can be treated as a classical when the wavelength of the mode is much larger than the horizon length. The Bogoliubov coefficients u and v can be calculated from eq. (2.18) where η_0 denotes the conformal time in a region in which the trans-Planckian effect becomes important, and η is the time after the mode leaves the trans-Planckian cutoff scale. We choose the WKB solution (2.28) as the preferred complex solution $\varphi_k(\eta_0)$ in eq. (2.19), which implies squeezing may occur through the trans-Planckian region. With the solutions (2.28) and (2.30) in each region, u and v take the form

$$u(\eta,\eta_0) = e^{-ik\eta + i\int\omega d\eta}\sqrt{\frac{k}{4\omega}}\left[1 + \frac{\omega}{k} - \frac{1}{k^2\eta_0\eta} - i\left(\frac{1}{k\eta_0} + \frac{\omega}{k^2\eta}\right)\right], \tag{2.38}$$

$$v(\eta,\eta_0) = e^{-ik\eta - i\int\omega d\eta}\sqrt{\frac{k}{4\omega}}\left[-1 + \frac{\omega}{k} + \frac{1}{k^2\eta_0\eta} + i\left(\frac{1}{k\eta_0} + \frac{\omega}{k^2\eta}\right)\right]. \tag{2.39}$$

3. Power spectrum with nonclassical states

In this section we calculate the power spectrum of gravitational wave perturbation when the perturbation mode stretches out to the Hubble radius during the inflation. The two-point correlation function at equal times

$$\langle\Psi|f(\eta,\mathbf{x})f(\eta,\mathbf{x}+\mathbf{r})|\Psi\rangle \equiv \int_0^\infty \frac{dk}{k}\frac{\sin kr}{kr}P_f(k)\,, \tag{3.1}$$

gives the power spectrum defined by

$$P_f(k) = \frac{k^3}{2\pi^2}\int d^3r e^{-i\mathbf{k}\cdot\mathbf{r}}\langle\Psi|f(\mathbf{x})f(\mathbf{x}+\mathbf{r})|\Psi\rangle \tag{3.2}$$

$$= \frac{k^3}{2\pi^2}\langle|f_k|^2\rangle\,. \tag{3.3}$$

Further, we consider the nonclassical states such as squeezed, squeezed coherent, and squeezed thermal states instead of the vacuum. Nonclassical states are quantum states that have no classical analog. These states have the quasiprobability distributions which may not always compatible with the interpretation as a probability distribution [13]. The quasiprobability distribution $W(\alpha,s)$ is used to show the nonclassical behavior quantitatively and the existence of negative regions of distribution for nonclassical states. The quasiprobability distribution, for example, the Glauber-Sudarshan $P(\alpha)$, is obtained from the complex Fourier transformation of eq. (1.2) [13]. In appendix A we briefly review the properties of the quasiprobability distribution, $W(\alpha,s)$.

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3.1 Squeezed state

First we consider the squeezed quantum state. In the previous section, we described the properties of the squeezed state. The ground squeezed state has a gaussian wave function

$$\Psi_0^{(s)}(\mu_k, \mu_{-k}, \eta) = \mathcal{N} \exp\left[-i\frac{\varpi_{ks}^*}{\varphi_{ks}^*}|\mu_k|^2\right], \tag{3.4}$$

which is derived from the fact that $\hat{a}_{ks}|0,\eta\rangle_s = 0$. Here $\mathcal{N}$ is a normalization factor. On the other hand, excited squeezed states (squeezed number state) have the non-gaussian wave functions [24]

$$\Psi_n^{(s)}(\mu_k, \mu_{-k}, \eta) = (-1)^n \left(\frac{\varphi_k}{\varphi_k^*}\right)^n L_n\left(\frac{|\mu_k|^2}{|\varphi_k|^2}\right)\Psi_0^{(s)}, \tag{3.5}$$

where L_n is a Laguerre polynomial. The temperature correlation functions of excited squeezed states show the nongaussianity.

To calculate the power spectrum with the squeezed quantum state, we use the following relations

$$\begin{aligned}
{}_s\langle \hat{a}_k \hat{a}_{k'}\rangle_s &= u_k v_k^*(1+2n_k)\delta(k+k'), && (3.6)\\
{}_s\langle \hat{a}^\dagger_{-k} \hat{a}^\dagger_{-k'}\rangle_s &= -u_k^* v_k(1+2n_k)\delta(k+k'), && (3.7)\\
{}_s\langle \hat{a}_k \hat{a}^\dagger_{-k'}\rangle_s &= |u_k|^2(1+n_k)\delta(k+k') + |v_k|^2 n_k \delta(k+k'), && (3.8)\\
{}_s\langle \hat{a}^\dagger_{-k} \hat{a}_{k'}\rangle_s &= |u_k|^2 n_k\delta(k+k') + |v_k|^2(1+n_k)\delta(k+k'), && (3.9)
\end{aligned}$$

where the inverse transformation of eq. (2.9) was used

$$\begin{aligned}
\hat{a}_k &= u\hat{a}_{ks} + v^*\hat{a}^\dagger_{-ks},\\
\hat{a}^\dagger_{-k} &= u^*\hat{a}^\dagger_{-ks} + v\hat{a}_{ks}.
\end{aligned} \tag{3.10}$$

Then the two point correlation function of h_{ij} with respect to the squeezed quantum state is

$$\begin{aligned}
{}_s\langle n,\eta|h_{ij}(\mathbf{x})h^{ij}(\mathbf{x}+\mathbf{r})|n,\eta\rangle_s &= \frac{1}{a^2}\int \frac{d^3k}{(2\pi)^{3/2}}\frac{d^3k'}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} e^{i\mathbf{k}'\cdot(\mathbf{x}+\mathbf{r})} \times \\
&\quad \times {}_s\langle n,\eta|\mu_k(\eta)\mu_{k'}(\eta)|n,\eta\rangle_s && (3.11)\\
&= \frac{1}{a^2}\int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}}|\varphi_k|^2(1+2n_k) \times \\
&\quad \times [|u_k|^2 + |v_k|^2 + 2\,\mathrm{Re}(u_k^* v_k)]\,. && (3.12)
\end{aligned}$$

From this two-point correlation function, we can read off the power spectrum of the gravitational wave

$$P_{gw}(k) = \frac{k^3}{2\pi^2}\frac{|\varphi_k|^2}{a^2}(1+2n_k)[(|u_k|^2 + |v_k|^2) + 2\,\mathrm{Re}(u_k^* v_k)]\,. \tag{3.13}$$

This gravitational wave power spectrum may be rewritten, using the squeezed mode φ_{ks} in eq. (2.19), as

$$P_{gw}(k) = \frac{k^3}{2\pi^2}\frac{|\varphi_{ks}|^2}{a^2}(1+2n_k). \tag{3.14}$$

For $\eta \geq \eta_c$, $\omega \simeq k$ and $k\eta_c = M/H$, one has

$$|u|^2 = \cosh^2 r \simeq 1 + \frac{H^4}{4M^4}\,,$$
$$|v|^2 = \sinh^2 r \simeq \frac{H^4}{4M^4}\,,$$
$$\mathrm{Re}(u^* v) = \frac{1}{2}\sinh 2r \cos\phi \simeq \frac{H^2}{2M^2}\cos\left(\frac{2M}{H}\right), \tag{3.15}$$

where we have used the mode crossing the cutoff scale at $\eta = \eta_c$. The growing mode solution of eq. (2.11) for $k \ll aH$ is given by $\varphi_k(\eta) \propto a(\eta)$. Thus, the resulting power spectrum for the superhorizon sized perturbations is

$$P_{gw}(k) = \left(\frac{H}{2\pi}\right)^2 (1+2n_k)\left[1 + \frac{H^2}{M^2}\cos\left(\frac{2M}{H}\right)\right]. \tag{3.16}$$

The power spectrum is corrected by the order H^2/M^2 and this result differs from ref. [25] but is similar to ref. [26] in order of magnitude. This correction is too small to be detected by the CMB observations, but linear order corrections in (H/M) are expected to be detectable [27]. We use the result of eq. (A.9) to show that the squeezed vacuum state $(n_k = 0)$ with the squeeze parameter in eq. (3.15) is a nonclassical state with the critical value (see appendix A)

$$s_c = e^{-2r} \simeq 1 - \frac{H^2}{M^2}\,. \tag{3.17}$$

This satisfies the condition for a state to be nonclassical $(0 \leq s_c \leq 1)$, which implies that the Glauber-Sudarshan P function $(s = 1)$ is more singular than a delta function.

We may consider a more general initial state in the trans-Planckian period given by a squeezed state which is obtained from eq. (2.15)

$$\varphi_{ks}(\eta_0) = u_0\varphi_k(\eta_0) + v_0\varphi_k^*(\eta_0)\,. \tag{3.18}$$

Here $\varphi_k(\eta_0)$ is the solution in eq. (2.28). Then the power spectrum for the superhorizon scales is

$$P_{gw}(k) \simeq \left(\frac{H}{2\pi}\right)^2 (1+2n_k)\Bigg[|u_0|^2 + |v_0|^2 + \frac{H^2}{M^2}\Bigg(|u_0|^2\cos\left(\frac{2M}{H}\right) + |v_0|^2\cos\left(\frac{2M}{H} - \phi_k\right) - {}$$
$$- 2|u_0||v_0|\cos\left(\frac{2M}{H} - 2\phi_k\right)\Bigg)\Bigg]. \tag{3.19}$$

The terms containing the squeeze phase ϕ_k may be set to zero due to the random character of the phase ϕ_k of each mode [28] to yield

$$P_{gw}(k) \simeq \left(\frac{H}{2\pi}\right)^2 (1+2n_k)\left[|u_0|^2 + |v_0|^2 + |u_0|^2\frac{H^2}{M^2}\cos\left(\frac{2M}{H}\right)\right]. \tag{3.20}$$

There is an increase of the overall factor by $|u_0|^2 + |v_0|^2$, which may be absorbed into the normalization of the spectrum. Once this normalization is done, the coefficient of the correction term can not be greater than one, thus slightly changing eq. (3.16). Note that the maximally squeezed states, $r_k \to \infty$, of the adiabatic vacuum have a factor 1/2.

3.2 Squeezed coherent state

Now we consider the squeezed coherent state. The squeezed coherent state is defined as an eigenstate of the annihilation operators $\hat{a}_{ks}(\eta)$ and $\hat{a}_{-ks}(\eta)$

$$\hat{a}_{ks}(\eta)|\alpha,\eta\rangle_s = \alpha_k|\alpha,\eta\rangle_s\,, \tag{3.21}$$

$$\hat{a}_{-ks}(\eta)|\alpha,\eta\rangle_s = \alpha_{-k}|\alpha,\eta\rangle_s\,, \tag{3.22}$$

where α_k and α_{-k} are complex constants, and

$$|\alpha,\eta\rangle_s = |\alpha_k,\alpha_{-k},z,\eta\rangle\,. \tag{3.23}$$

The squeezed coherent state can be treated algebraically by introducing a displacement operator

$$\begin{aligned}\hat{D}(\alpha) &\equiv \hat{D}(\alpha_k)\hat{D}(\alpha_{-k})\,,\\ &= e^{\alpha_k\hat{a}^\dagger_{ks}(\eta)-\alpha^*_k\hat{a}_{ks}(\eta)}e^{\alpha_{-k}\hat{a}^\dagger_{-ks}(\eta)-\alpha^*_{-k}\hat{a}_{-ks}(\eta)}\,,\end{aligned} \tag{3.24}$$

which is unitary

$$\hat{D}(\alpha)\hat{D}^\dagger(\alpha) = \hat{D}^\dagger(\alpha)\hat{D}(\alpha) = \hat{I}\,. \tag{3.25}$$

The displacement operator translates by constant amounts the annihilation and creation operators through the unitary transformation

$$\begin{aligned}\hat{D}^\dagger(\alpha)\hat{a}_{ks}(\eta)\hat{D}(\alpha) &= \hat{a}_{ks}(\eta)+\alpha_k\,,\\ \hat{D}^\dagger(\alpha)\hat{a}_{-ks}(\eta)\hat{D}(\alpha) &= \hat{a}_{-ks}(\eta)+\alpha_{-k}\,.\end{aligned} \tag{3.26}$$

Similar relations hold for $\hat{a}^\dagger_{ks}$ and $\hat{a}^\dagger_{-ks}$. Hence one can see that the squeezed coherent state results from the unitary transformation of the squeezed state

$$|\alpha,\eta\rangle_s = \hat{D}(\alpha)|\eta\rangle_s = \hat{D}(\alpha)\hat{S}(z,\eta)|\eta\rangle\,. \tag{3.27}$$

The expectation values of $\hat{\mu}_k$ and $\hat{\pi}_k$ with respect to the squeezed coherent state are

$$\begin{aligned}\mu_{kc} &\equiv {}_s\langle\alpha,\eta|\hat{\mu}_k|\alpha,\eta\rangle_s = {}_s\langle\alpha,\eta|(\hat{a}_k\varphi_k+\hat{a}^\dagger_{-k}\varphi^*_{-k})|\alpha,\eta\rangle_s\\ &= (u_k\alpha_k+v^*_{-k}\alpha^*_{-k})\varphi_k+(u^*_{-k}\alpha^*_{-k}+v_k\alpha_k)\varphi^*_{-k}\,,\end{aligned} \tag{3.28}$$

$$\begin{aligned}\pi_{kc} &\equiv {}_s\langle\alpha,\eta|\hat{\mu}_k|\alpha,\eta\rangle_s = {}_s\langle\alpha,\eta|\left(\hat{\mu}'_k-\frac{a'}{a}\hat{\mu}_k\right)|\alpha,\eta\rangle_s\\ &= \mu'_{kc}-\frac{a'}{a}\mu_{kc}\,.\end{aligned} \tag{3.29}$$

Note that $\mu_{kc}(\eta)$ is the solution of eq. (2.11).

We calculate the following relations with respect to the squeezed coherent state

$$\begin{aligned}{}_s\langle\alpha,\eta|\hat{a}_k\hat{a}_{k'}|\alpha,\eta\rangle_s &= [u_k^2\alpha_k^2+{v^*_{-k}}^2{\alpha^*_{-k}}^2]\delta(k-k')+u_kv^*_k\delta(k+k')+\\ &\quad+2u_kv^*_{-k}\alpha^*_{-k}\alpha_k\delta(k-k')\,,\end{aligned} \tag{3.30}$$

$$
\begin{aligned}
{}_s\langle\alpha,\eta|\hat a^\dagger_{-k}\hat a^\dagger_{-k'}|\alpha,\eta\rangle_s &= [u^*_{-k}{}^2\alpha^*_{-k}{}^2 + v_k^2\alpha_k^2]\delta(k-k') + u_k^*v_k\delta(k+k') + \\
&\quad +2u^*_{-k}v_k\alpha^*_{-k}\alpha_k\delta(k-k')\,, \qquad (3.31)\\
{}_s\langle\alpha,\eta|\hat a_k\hat a^\dagger_{-k'}|\alpha,\eta\rangle_s &= |u_k|^2\delta(k+k') + (u_ku^*_{-k} + v_kv^*_{-k})\alpha_k\alpha^*_{-k}\delta(k-k') + \\
&\quad +[u_kv_k\alpha_k^2 + u^*_{-k}v^*_{-k}\alpha^*_{-k}{}^2]\delta(k-k')\,, \qquad (3.32)\\
{}_s\langle\alpha,\eta|\hat a^\dagger_{-k}\hat a_{k'}|\alpha,\eta\rangle_s &= |v_k|^2\delta(k+k') + (u^*_{-k}u_k + v^*_{-k}v_k)\alpha^*_{-k}\alpha_k\delta(k-k') + \\
&\quad +(u_kv_k\alpha_k^2 + u^*_{-k}v^*_{-k}\alpha_{-k}{}^2)\delta(k-k')\,. \qquad (3.33)
\end{aligned}
$$

With these relations, we find the two point correlation function of h_{ij} with respect to the squeezed coherent state

$$
\begin{aligned}
{}_s\langle\alpha,\eta|h_{ij}(\mathbf{x})h^{ij}(\mathbf{x}+\mathbf{r})|\alpha,\eta\rangle_s &= \frac{1}{a^2}\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot\mathbf{r}}\times\\
&\quad\times[|\varphi_k|^2\{(|u_k|^2+|v_k|^2) + 2\,\mathrm{Re}(u_k^*v_k)\} + e^{2i\mathbf{k}\cdot\mathbf{x}}\mu_{kc}^2]\,. \qquad (3.34)
\end{aligned}
$$

We finally obtain the power spectrum

$$
P_{gw} = \frac{k^3}{2\pi^2a^2}[|\varphi_k|^2\{(|u_k|^2+|v_k|^2) + 2\,\mathrm{Re}(u_k^*v_k)\} + e^{2i\mathbf{k}\cdot\mathbf{x}}\mu_{kc}^2]\,. \qquad (3.35)
$$

The power spectrum consists of two parts; one is the contribution from the squeezed vacuum state as in eq. (3.13) for $n_k = 0$ and the other is a classical part which is the expectation value with respect to the squeezed coherent state. The classical part μ_{kc} satisfies the classical equation of motion (2.8). As shown in eq. (A.9), the regularity condition does not depend on the amplitude, α, of the squeezed coherent states so these states also satisfy this condition as in the case of squeezed vacuum states.

3.3 Squeezed thermal state

We now use thermofield dynamics (TFD) for time-dependent system [29, 30] and calculate the power spectrum of gravitational wave in a squeezed thermal state. The thermofield dynamics is a canonical formalism for finite temperature theory to describe quantum systems in thermal equilibrium. The idea of TFD is to double the system by adding a fictitious system and extend the thermal equilibrium in the system's Hilbert space to a thermal state, a pure state, in the extended Hilbert space of the total system. We briefly introduce the TFD formalism in one-mode state and after that calculate the power spectrum using the two-mode state TFD formalism. The density operators are defined by

$$
\begin{aligned}
\rho_k(\eta) &= \frac{1}{Z}e^{-\beta\omega_k\hat a_k^\dagger(\eta)\hat a_k(\eta)}\,,\\
\tilde\rho_k(\eta) &= \frac{1}{Z}e^{-\beta\omega_k\tilde{\hat a}_k^\dagger(\eta)\tilde{\hat a}_k(\eta)}\,, \qquad (3.36)
\end{aligned}
$$

where

$$
Z = \mathrm{Tr}\,e^{-\beta\hat{\mathcal{H}}_k}\,. \qquad (3.37)
$$

Here β and ω are constants that may be fixed by the initial temperature and frequency. The density operator in the extended Hilbert space is given by

$$
\hat\rho_k(\eta) = \rho_k(\eta)\otimes\tilde\rho_k(\eta) = \frac{1}{Z^2}e^{-\beta\omega_k(\hat a_k^\dagger(\eta)\hat a_k(\eta) - \tilde{\hat a}_k^\dagger(\eta)\tilde{\hat a}_k(\eta))}\,. \qquad (3.38)
$$

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The thermal expectation value of an operator $\hat{A}$ of the system now takes the form

$$\langle \hat{A} \rangle = \mathrm{Tr}\, \rho_k(\eta) \hat{A} = \langle 0(\beta), \eta | \hat{A} | 0(\beta), \eta \rangle\,, \tag{3.39}$$

where the thermal vacuum state is given by

$$|0(\beta), \eta\rangle = \sqrt{1 - e^{-\beta\omega_k}}\, e^{-(\beta\omega_k/2)\hat{a}_k^\dagger(\eta)\tilde{\hat{a}}_k^\dagger(\eta)} |0, \eta\rangle\,, \tag{3.40}$$

with $|0, \eta\rangle = |0, \tilde{0}, \eta\rangle$. As $\{\hat{a}_k, \hat{a}_k^\dagger\}$ and $\{\tilde{\hat{a}}_k, \tilde{\hat{a}}_k^\dagger\}$ describe two independent systems, the nonzero commutation relations become

$$[\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta(k - k')\,, \qquad [\tilde{\hat{a}}_k, \tilde{\hat{a}}_{k'}^\dagger] = \delta(k - k')\,. \tag{3.41}$$

The thermal state is written as a time-dependent squeezed state of the vacuum state

$$|0(\beta), \eta\rangle = \hat{T}(\theta) |0, \eta\rangle\,, \tag{3.42}$$

where

$$\hat{T}(\theta) = \exp[-\theta_k(\beta)\{\tilde{\hat{a}}_k(\eta)\hat{a}_k(\eta) - \hat{a}_k^\dagger(\eta)\tilde{\hat{a}}_k^\dagger(\eta)\}]\,. \tag{3.43}$$

Here $\theta_k(\beta)$ is a temperature-dependent parameter determined by

$$\begin{aligned} \cosh\theta_k(\beta) &= (1 - e^{-\beta\omega_k})^{-1/2}\,, \\ \sinh\theta_k(\beta) &= e^{-\beta\omega_k/2}(1 - e^{-\beta\omega_k})^{-1/2}\,. \end{aligned} \tag{3.44}$$

In the two-mode state formalism, the thermal operator $\hat{T}(\theta)$ can be written as

$$\hat{T}(\theta) = \exp[-\theta_k(\beta)\{\hat{a}_{ks}\tilde{\hat{a}}_{ks} - \hat{a}_{ks}^\dagger\tilde{\hat{a}}_{ks}^\dagger\}]\exp[-\theta_k(\beta)\{\hat{a}_{-ks}\tilde{\hat{a}}_{-ks} - \hat{a}_{-ks}^\dagger\tilde{\hat{a}}_{-ks}^\dagger\}]\,. \tag{3.45}$$

Then the time- and temperature-dependent annihilation and creation operators through the Bogoliubov transformation become

$$\hat{a}_k(\beta, \eta) = \hat{T}(\theta)\hat{a}_{ks}\hat{T}^\dagger(\theta) = \cosh\theta_k(\beta)\hat{a}_{ks}(\eta) - \sinh\theta_k(\beta)\tilde{\hat{a}}_{-ks}^\dagger(\eta)\,, \tag{3.46}$$

$$\tilde{\hat{a}}_k(\beta, \eta) = \hat{T}(\theta)\tilde{\hat{a}}_{ks}\hat{T}^\dagger(\theta) = \cosh\theta_k(\beta)\tilde{\hat{a}}_{ks}(\eta) - \sinh\theta_k(\beta)\hat{a}_{-ks}^\dagger(\eta)\,. \tag{3.47}$$

We get similar equations for $\hat{a}_k^\dagger(\beta, \eta), \tilde{\hat{a}}_k^\dagger(\beta, \eta)$ by using the hermitean conjugate of these equations. The thermal state is a time- and temperature-dependent vacuum

$$\hat{a}_k(\beta, \eta)|0(\beta), \eta\rangle_s = \tilde{\hat{a}}_k(\beta, \eta)|0(\beta), \eta\rangle_s = 0\,. \tag{3.48}$$

The thermal state $|0(\beta), \eta\rangle_s$, as an eigenstate of the invariant operators $\hat{a}_k(\beta, \eta)$, is an exact eigenstate of the total system. To fix ω_k in eq. (3.36), we assume that at $\eta = \eta_i$ the universe was in thermal equilibrium state. And if we neglect the expansion of the spacetime at $\eta = \eta_i$, the hamiltonian is approximated as

$$\mathcal{H}_k = \pi_k\pi_{-k} + \omega_k\mu_k\mu_{-k}\,, \tag{3.49}$$

to which we can set $\omega_k = a(\eta_i)\omega_{\mathrm{phys}}(k/a(\eta_i))$.

Using a similar method as in the previous sections, we get the following relations

$$ {}_s\langle\theta(\beta),\eta|\hat{a}_k\hat{a}_{k'}|\theta(\beta),\eta\rangle_s = u_k v_k^*(\cosh^2\theta_k + \sinh^2\theta_k)\delta(k+k') \,, \tag{3.50} $$

$$ {}_s\langle\theta(\beta),\eta|\hat{a}^\dagger_{-k}\hat{a}^\dagger_{-k'}|\theta(\beta),\eta\rangle_s = u_k^* v_k(\cosh^2\theta_k + \sinh^2\theta_k)\delta(k+k') \,, \tag{3.51} $$

$$ {}_s\langle\theta(\beta),\eta|\hat{a}_k\hat{a}^\dagger_{-k'}|\theta(\beta),\eta\rangle_s = (|u_k|^2\cosh^2\theta_k + |v_k|^2\sinh^2\theta_k)\delta(k+k') \,, \tag{3.52} $$

$$ {}_s\langle\theta(\beta),\eta|\hat{a}^\dagger_{-k}\hat{a}_{k'}|\theta(\beta),\eta\rangle_s = (|u_k|^2\sinh^2\theta_k + |v_k|^2\cosh^2\theta_k)\delta(k+k') \,. \tag{3.53} $$

Using the above relations, we can find the two-point correlation functions with the squeezed thermal state

$$ \begin{aligned} {}_s\langle\theta(\beta),\eta|h_{ij}(\mathbf{x})h^{ij}(\mathbf{x}+\mathbf{r})|\theta(\beta),\eta\rangle_s &= \frac{1}{a^2}\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot\mathbf{r}}|\varphi_k|^2[(|u_k|^2+|v_k|^2)+2\,\mathrm{Re}(u_k v_k)]\times \\ &\quad\times(1+2\sinh^2\theta_k) \,, \end{aligned} \tag{3.54} $$

from which we finally obtain the power spectrum

$$ P_{gw}(k) = \frac{k^3}{2\pi^2}\frac{|\varphi_k|^2}{a^2}[(|u_k|^2+|v_k|^2)+2\,\mathrm{Re}(u_k v_k)](1+2\sinh^2\theta_k) \,. \tag{3.55} $$

Here θ_k follows from eq. (3.44) as determined by the Bose-Einstein distribution at the initial time:

$$ \sinh^2\theta_k = \frac{1}{e^{\beta\omega_k}-1} \,. \tag{3.56} $$

Thus the resulting power spectrum with the squeezed thermal state is corrected by the thermal contribution factor $(1+2\sinh^2\theta_k)$. The condition of the quasiprobability distribution $W(\alpha,s)$ being regular for the squeezed thermal state is from (A.9),

$$ s < s_c = \frac{1}{1-e^{-\beta\omega_k}}\left(1-\frac{H^2}{M^2}\right) , \tag{3.57} $$

where we have used the relations (3.15). In the limit of zero temperature, $\beta\to\infty$, this condition reduces to that of the squeezed vacuum state. However, in the high temperature limit, $\beta\to 0$, this state may be regarded as a classical state.

4. Conclusion and discussion

We have studied the spectrum of gravitational wave perturbations in nonclassical states during inflation. Gravitational wave or primordial density perturbation, which is responsible for the large scale structure formation, would have been generated from quantum fluctuations when the wavelengths of the perturbation modes were much smaller than the horizon scales. After the wavelengths were stretched out to the horizon scale during inflation, they could be treated as a classical perturbation.

Any state can be, in the cosmological expansion, highly squeezed and this phenomenon is quite a quantum effect which cannot be explained in classical theory. Squeezing can occur when the time-dependent parameter is amplified at the subhorizon scale. We considered in this paper that an initial state can evolve to its highly squeezed state due to the amplifying time-dependent parameter which plays a role of the modified dispersion relation in trans-Planckian physics. So the gravitational field may exist, in general, in a squeezed quantum state during inflation. This is a dynamical process occurring through eq. (2.19). In eq. (2.19), the Bogoliubov coefficients u and v are functions of η and η_0 and satisfy the eq. (2.11). From this point of view, the underlying physical concept differs from that of ref. [8, 9]. Actually in ref. [8, 9], since the squeezed state was put as an initial condition, there was not a dynamical squeezing process. On the contrary, in this paper any initial state is squeezed through the dynamical process due to the time-dependent parameter.

We also considered the nonclassical state whose behavior can be quantitatively described by the quasiprobability distribution such as squeezed quantum state, squeezed coherent state, and squeezed thermal state. The condition for the quasiprobability distribution to be regular is used to test whether the states are either classical or nonclassical.

To compare with the observation, we calculated the power spectrum of the gravitational wave perturbation with the nonclassical states from the two-point correlation function. For the squeezed state, the power spectrum is corrected by $(H/M)^2$ which is too small to be detected from the observation. It is expected for linear order, H/M, corrections to be detectable by the observations [27]. And the squeeze parameter satisfies the condition for the states being nonclassical. The power spectrum for the squeezed coherent state consists of squeezed vacuum state contribution part and classical part. The classical quantity μ_{kc}, which is the expectation value of the gravitational field with respect to the squeezed coherent state, satisfies the classical equation of motion (2.8). The regular condition of quasiprobability distribution of the squeezed coherent state does not depend on the amplitude of the squeezed coherent state, so this state may be nonclassical as the squeezed vacuum state. And for the squeezed thermal state, the power spectrum is modified by the thermal contribution $(1 + 2\sinh^2\theta(\beta_k))$. The nonclasscial condition depends on the parameter β and ω_k.

Finally, the quasiprobability distribution measures the degree of the nonclassical behavior of states. This property will be used to investigate the transition from quantum to classical perturbations during the inflation for superhorizon scales.

Acknowledgments

We would like to thank D.N. Page and D. Pogosyan for useful discussions. We also would like to appreciate the warm hospitality of the Theoretical Physics Institute, University of Alberta. This was supported by Korea Astronomy Observatory (KAO).

A. Quasiprobability distribution

We briefly review the properties of the quasiprobability distribution [13]. The ensemble average of the operator F is represented by the density operator

$$\langle F\rangle = \mathrm{Tr}(\rho F)\,. \tag{A.1}$$

This can be expressed in terms of the weight function $W(\alpha, s)$ by the integral form

$$\mathrm{Tr}(\rho F) = \frac{1}{\pi} \int d^2\alpha W(\alpha, s) f(\alpha, -s)\,, \tag{A.2}$$

where α is a complex variable. Though $W(\alpha, s)$ can not exactly be interpreted as a probability distribution, it plays so closely the role of one that it is referred to as a quasiprobability distribution. The $W(\alpha, s)$ is identified with Glauber-Sudarshan (P), with the Wigner (W), and with Husimi (Q) distribution corresponding to the values $s = 1, 0,$ and -1, respectively [31].

Through complex Fourier transformations [32], $W(\alpha, s)$ is expressed in terms of the characteristic function $\chi(\xi, s)$ as

$$W(\alpha, s) = \frac{1}{\pi} \int e^{\alpha\xi^* - \alpha^*\xi} \chi(\xi, s) d^2\xi\,, \tag{A.3}$$

where

$$\chi(\xi, s) = \mathrm{Tr}(\rho \hat{D}(\xi)) e^{\frac{s}{2}|\xi|^2}\,, \tag{A.4}$$

with the displacement operator $\hat{D}(\xi)$. The $W(\alpha, s)$ is normalized to unit through the trace of density operator

$$1 = \mathrm{Tr}\,\rho = \frac{1}{\pi} \int W(\alpha, s) d^2\alpha\,, \tag{A.5}$$

and satisfies the Hermiticity

$$W(\alpha, s^*) = W(\alpha, s)^*\,, \tag{A.6}$$

which shows that the function $W(\alpha, s)$ is real for real values s. Especially, the P distribution is more useful in measuring the degree of the nonclassical behavior. If the P distribution of any state is more singular than a delta function and is not positive definite, this state is classified as a nonclassical state [15]. The P distribution no more singular than a delta function will be said to regular.

The characteristic function of the squeezed coherent thermal states is derived [33]

$$\chi(\xi, s) = \exp\left[-A|\xi|^2 - \frac{1}{2}(B^*\xi^2 + B\xi^{*2}) + C^*\xi - C\xi^*\right], \tag{A.7}$$

where

$$\begin{aligned} A &= \frac{1-s}{2} + \bar{n} + (2\bar{n}+1)\sinh^2 r\,, \\ B &= -(2\bar{n}+1)e^{i\phi} \sinh r \cosh r\,, \\ C &= \lambda\,, \\ \bar{n} &= (e^{\beta\omega} - 1)^{-1}\,. \end{aligned} \tag{A.8}$$

The condition for the integration of the quasiprobability distribution (A.3) to be regular with the characteristic function (A.7) is

$$s < s_c \equiv (2\bar{n}+1)e^{-2r}\,. \tag{A.9}$$

Then the quasiprobability distribution is calculated for squeezed coherent thermal states [33]

$$W(\alpha, s) = (A^2 - |B|^2)^{-\frac{1}{2}} \times \exp\left[-\frac{1}{A^2 - |B|^2}\left(A|\alpha - C|^2 + \frac{1}{2}[B^*(\alpha - C)^2 + B(\alpha^* - C^*)]\right)\right]. \tag{A.10}$$

This result can be applied to the squeezed vacuum state by letting $\lambda = 0$ and $\bar{n} = 0$, to the squeezed coherent state by $\bar{n} = 0$, and to the squeezed thermal state by $\lambda = 0$. Depending on whether a state satisfies the condition (A.9), it shows either a classical or nonclassical behavior. This result simply extends to the two-mode squeezed state if the characteristic function is given by the exponential of a quadratic form [31, 35].

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